## Heterogeneity on the Road

Meeting on Modeling, Analysis, Control

February 1-2, 2024

# Contents

Titles and Abstracts	1
Hard congestion limit of one-dimensional Euler equations with singular	
pressure in the BV setting (Fabio Ancona)	1
C-ITS Services network along the A4 and A57 motorways (Nicola Busatto)	1
Multi-scale and ML-aided models for vehicular traffic: A Collaboration	
with Autostrade Alto Adriatico (Emiliano Cristiani, Elia Onofri)	1
The Air Traffic Flow Management problem: Integer Programming ap-	
$proaches (Luigi De Giovanni) \dots \dots$	1
Traffic Control: a Multi-Scale Perspective (Antonella Ferrara)	1
Multi-class and multi-population traffic flow models on networks (Paola	
<i>Goatin</i> )	2
Traffic flow characterization in the era of Big Data: current trends and	
challenges ( <i>Ibai Lana</i> )	2
A business perspective on modern mobility challenges (Marco Pezzulla).	2
Smoothing traffic via autonomous vehicles and control of large networked	
systems (Benedetto Piccoli)	3
Overview on conservation laws with point constraints (Massimiliano Rosini)	3
Topological interactions: a walk through friendship in mathematics (Francesco	
	3
Regularity and control for conservation laws with discontinuous flux (Luca	
<i>Talamini</i> )	4

List of participants

## **Titles and Abstracts**

## Hard congestion limit of one-dimensional Euler equations with singular pressure in the BV setting

Fabio Ancona

Università di Padova

## C-ITS Services network along the A4 and A57 motorways

Nicola Busatto Concessioni Autostradali Venete S.p.A.

## Multi-scale and ML-aided models for vehicular traffic: A Collaboration with Autostrade Alto Adriatico

Emiliano Cristiani, Elia Onofri CNR - Roma

## The Air Traffic Flow Management problem: Integer Programming approaches

Luigi De Giovanni Università di Padova

## Traffic Control: a Multi-Scale Perspective

Antonella Ferrara Università di Pavia

## Multi-class and multi-population traffic flow models on networks

Paola Goatin

#### INRIA Sophia Antipolis - Méditeranée

We present a general framework for the modeling and simulation of multi-class (i.e. different vehicle types) and multi-population (i.e. different routes) traffic flow on road networks. The models consist of systems of conservation laws coupled in the speed component of the flux function, allowing for overtaking and creeping of different vehicle types. Suitable, class specific, coupling conditions at junctions allow to design Godunov type numerical schemes exploiting the supply-demand formulation. The simulation can then be coupled to routing strategies at road junctions to represent the choices of the different populations of drivers. Some numerical tests are presented to illustrate the model behaviours.

# Traffic flow characterization in the era of Big Data: current trends and challenges

Ibai Lana Tecnalia, BRTA Derio, Spain

### A business perspective on modern mobility challenges

Marco Pezzulla

Umovity, Roma

The landscape of mobility is undergoing transformative changes that significantly impact businesses across diverse sectors. This presentation aims to identify possible commonalities between academic research and challenges arising from the mobility market. The session will begin with a brief introduction to the Umovity company and its connection to the academic world. Delving into use cases and solutions for both technical and theoretical problems, we will illustrate how a company navigates the complexities of the industry.

Finally, the focus will shift to open problems, potential starting points for future collaborations, and an exploration of why Machine Learning (ML) and Artificial Intelligence (AI) are so appealing for the mobility market. We will highlight key factors that contribute to the rapid growth of these fields in modern software over the years.

## Smoothing traffic via autonomous vehicles and control of large networked systems

Benedetto Piccoli Rutgers University

The problem of control of large multi-agent systems is attracting attention for the many possible applications, such as crowd dynamics, socio-economic systems, and vehicular traffic. After revising some challenges and opportunities, we will focus on a recent experiment involving 100 autonomous vehicles to dampen stop-and-go waves on an open highway.

## Overview on conservation laws with point constraints

Massimiliano Rosini University of Chieti - Pescara

In this presentation, we provide a comprehensive overview of significant advancements in the study of conservation laws featuring variable unilateral point constraints on the flow or velocity. Notably, this research has practical applications in diverse fields such as vehicular traffic, pedestrian movements, and gas flows.

Commencing with a concise historical review, we trace the evolution of the theory of point constraints applied to Lighthill-Whitham-Richards (LWR), Aw-Rascle-Zhang (ARZ), and phase transition (PT) models. Our exploration encompasses a detailed examination of the construction of local point constraints for the LWR model, both at the macroscopic and microscopic levels. Throughout this analysis, we highlight the limitations inherent in such models and elucidate the rationales behind various generalizations found in the existing literature.

Lastly, we briefly demonstrate adaptations of this theory to both the ARZ and PT models, offering an insightful glimpse into how these results can be applied beyond the LWR model.

## Topological interactions: a walk through friendship in mathematics

Francesco Rossi

IUAV - Venezia

I will first show several non-mathematical instances of the concept of friend, or acquaintance. I will then present a corresponding mathematical model, introducing topological interactions.

I will show both ordinary and partial differential equations for topological interactions and provide new results about the structure of solutions. Finally, I will discuss an open problem about the mean-field limit of this model.

# Regularity and control for conservation laws with discontinuous flux

Luca Talamini

Università di Padova

We consider the Cauchy problem for the scalar conservation law

$$u_t + f(u, x)_x = 0 \tag{1}$$

where f is a discontinuous function

$$f(u, x) = \begin{cases} f_l(u), & x < 0, \\ f_r(u), & x > 0 \end{cases}$$

Here  $f_l$ ,  $f_r$  are strictly convex maps. Conservation laws with discontinuous flux have numerous applications; two well known examples are traffic flow with heterogeneus road conditions and two phase flow in porous media. The discontinuity of the flux naturally leads to the study of infinitely many  $L^1$  contractive semigroups  $S_t^{AB}$ , each one associated to particular pair of values, a *connection*, (A, B). The solution uassociated to (A, B) will be the unique one that dissipates the additional generalized Kružkov entropy

$$\eta^{AB} = \begin{cases} |u - A|, & x < 0, \\ |u - B|, & x > 0 \end{cases}$$

In general solutions of (1) do not have bounded total variation near the interface x = 0. Motivated by the study of regularity near the interface and by controllability problems, we shall prove some adapted Oleinik estimates and illustrate some extensions on networks of roads ("junction problems").

The second part of the talk will be devoted to the problem of initial data identification. In particular, for  $\omega \in L^{\infty}$ , our goal will be to characterize the set

$$\mathcal{I}_T^{AB}\omega = \{u_0 \in \mathbb{L}^\infty : \mathcal{S}_T^{AB}u_0 = \omega\}$$

In particular, we shall prove that  $\mathcal{I}_T^{AB}\omega$  is either empty, a singleton, or an infinite dimensional (non convex) cone. Finally, we shall define an appropriate notion of backward operator through which the vertex of the cone  $\mathcal{I}_T^{AB}\omega$  can be characterized.

# List of participants

- Fabio Ancona (Università di Padova)
- Martino Bardi (Università di Padova)
- Mohamed Bentaibi (Università di Padova)
- Roberta Bianchini (CNR Roma)
- Stefano Bianchini (SISSA Trieste)
- Maya Briani (CNR Roma)
- Nicola Busatto (Concessioni Autostradali Venete)
- Laura Caravenna (Università di Padova)
- Annalisa Cesaroni (Università di Padova)
- Alexander Cliffe (Università di Padova)
- Emiliano Cristiani (CNR Roma)
- Luigi De Giovanni (Università di Padova)
- Carlotta Donadello (Université Franche Comté)
- Enrico Ferrante (Società Autostrade Alto Adriatico)
- Antonella Ferrara (Università di Pavia)
- Mauro Garavello (Università di Milano Bicocca)
- Paola Goatin (INRIA Sophia Antipolis)
- Ibai Lana (TECNALIA)
- Claudio Marchi (Università di Padova)
- Elio Marconi (Università di Padova)
- Andrea Marson (Università di Padova)
- Elia Onofri (CNR Roma)

- Marco Pezzulla (UMOVITY)
- Massimiliano Rosini (Università di Chieti Pescara)
- Francesco Rossi (IUAV Venezia)
- Laura Spinolo (CNR Pavia)
- Luca Talamini (Università di Padova)
- Davide Vittone (Università di Padova)