UNIVERSITÀ DEGLI STUDI DELL'AQUILA

DISIM
Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica

PRIN 2020

BOOK OF ABSTRACTS AND CONFERENCE SCHEDULE

12TH MEETING ON NONLINEAR EVOLUTION PDES, FLUID DYNAMICS, AND TRANSPORT EQUATIONS

L'AQUILA, 13-15 JULY, 2022

Department of Information Engineering, Computer Science and Mathematics
University of L'Aquila

Website: https://sites.google.com/view/12thmenlevpdefdte/home

ORGANIZING COMMITTEE

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Stefano Spirito (Università degli Studi dell'Aquila)

LIST OF SPEAKERS

Paolo Antonelli (Gran Sasso Science Institute)
Roberta Bianchini (IAC-CNR Roma)
Elisabetta Chiodaroli (Università di Pisa)
Giuseppe Maria Coclite (Politecnico di Bari)
Sara Daneri (Gran Sasso Science Institute)
Michele Dolce (Imperial College London)
Simone Fagioli (Università degli Studi dell'Aquila)
Mauro Garavello (Università degli Studi di Milano-Bicocca)
Graziano Guerra (Università degli Studi di Milano-Bicocca)
Giacomo Leccese (Scuola Internazionale Superiore di Studi Avanzati, Trieste)
Alessandro Morando (Università degli Studi di Brescia)
Laura Spinolo (IMATI-CNR Pavia)
Stefano Spirito (Università degli Studi dell'Aquila)
Luca Talamini (Università degli Studi di Padova)
Martina Zizza (Scuola Internazionale Superiore di Studi Avanzati, Trieste)

SCHEDULE

- **WEDNESDAY, 13 July**
  - 14:00-14:20. Registration
  - 14:20-14:30. Opening
  - 14:30-15:00. **Coclite**: *Nonlocal regularization of conservation laws*
  - 15:05-15:35. **Guerra**: *Balance laws with singular source term and applications to fluid dynamics*
  - 15:40-16:10. Coffee Break
  - 16:10-16:40. **Garavello**: *Macroscopic traffic models and autonomous vehicles*
  - 16:45-17:15. **Bianchini**: *On the hydrostatic limit of stably stratified fluids*

- **THURSDAY, 14 July. Morning Session**
  - 9:30-10:00. **Morando**: *Local existence of 2D compressible current-vortex sheets*
  - 10:05-10:35. **Dolce**: *On maximally mixed equilibria of two-dimensional perfect fluids*
  - 10:40-11:10. Coffee Break
  - 11:10-11:40. **Daneri**: *On the sticky particle solutions to the pressureless Euler system in general dimension*
  - 11:45-12:15. **Spinolo**: *The singular local limit of nonlocal traffic models with general kernels*
  - 12:20-14:30. Lunch

- **THURSDAY, 14 July. Afternoon Session**
  - 14:30-15:00. **Spirito**: *Propagation of regularity and uniqueness for a Kelvin-Voigt model in viscoelasticity*
  - 15:05-15:35. **Leccese**: *On the sticky particle solutions to the multi-dimentional Hamiltonian system*
  - 15:40-16:00. Coffee Break
  - 16:00-16:30. **Talamini**: *Regularity and initial data identification for conservation laws with space discontinuous flux*
  - 16:35-17:05. **Zizza**: *Properties of mixing BV vector fields*
  - 17:10-18:10. Discussion on the PRIN Project

- **FRIDAY, 15 July**
  - 10:05-10:35. **Antonelli**: *Almost finite energy solutions for a quantum Euler-Maxwell system*
  - 10:40-11:10. Coffee Break
  - 11:10-11:40. **Chiodaroli**: *On wild initial data for the isentropic Euler system of gas dynamics*
  - 11:45-12:15. **Fagioli**: *On a chemotaxis-hapotaxis system with nonlinear diffusion modelling multiple sclerosis*
  - 12:20-12:30. Closing

Social dinner on Thursday 14 July 20:30,
at the restaurant Lo Scalco dell'Aquila

ABSTRACTS

Paolo Antonelli

Gran Sasso Science Institute

Almost finite energy solutions for a quantum Euler-Maxwell system

Abstract. In this talk I will present some recent results obtained in collaboration with Pierangelo Marcati and Raffaele Scandone about the existence of weak solutions for the quantum magnetohydrodynamic (QMHD) system in the three dimensional space. The QMHD system describes a charged quantum fluid interacting with its self-generated electromagnetic field. It is a prototypical model for a quantum plasma, arising for instance in the description of dense astrophysical objects such as white dwarf stars. In particular, the introduction of the quantum term is motivated by the fact that in such contexts the thermal de Broglie wavelength becomes comparable (larger than or equal) to the typical interatomic distance. As for many quantum fluid models, it is possible to establish an analogy between QMHD and an underlying wave function dynamics, which in this case is given by a nonlinear Maxwell-Schrödinger (MS) system. more precisely, the nonlinear MS system describes the coupled evolution of the order parameter and the electromagnetic potentials, which determine the fluid dynamical quantities and electro-magnetic fields in the QMHD system. We prove the existence of global in time, almost finite energy weak solutions, in the sense that the initial data related to the electromagnetic part are required to be slightly more regular than just finite energy. The main difficulty to overcome here is the lack of a satisfactory well-posedness theory for the nonlinear MS system. In fact, by avoiding any sort of regularizing argument in the derivation of the hydrodynamical equations, I will show that weak solutions to nonlinear MS system determine weak solutions to QMHD system. Once the main strategy is outline, I will then present the main a priori estimates needed to rigorously justify all passages. Moreover, I will also show a stability estimate for the hydrodynamic state and the Lorentz force. The proof of this result is based on a non-trivial extension of a Koch-Tzvetkov type smoothing estimate for the linear magnetic Schrödinger evolution, which then yields suitable local smoothing estimates for the nonlinear evolution.

References

[1] P. Antonelli, P. Marcati, R. Scandone, *Existence and Stability of almost finite energy weak solutions to the Quantum Euler-Maxwell system*,

arXiv:2109.14588 (2021).

Roberta Bianchini

IAC, Consiglio Nazionale delle Ricerche

On the hydrostatic limit of stably stratified fluids

Abstract. This talk will address the rigorous justification of the hydrostatic limit for continuously stratified incompressible fluids under the influence of gravity. The main peculiarity of this work with respect to previous studies is that no (regularizing) viscosity contribution is added to the fluid-dynamics equations and only diffusivity effects are included. More precisely, the diffusivity effects that we include are induced by an advection term whose specific form was proposed by Gent and McWilliams in the 90's to model effective eddy correlations for non-eddy-resolving systems, motivated by applications in oceanography.

This is a joint work with Vincent Duchêne (CNRS & IRMAR, Rennes).

References

[1] R. Bianchini, V. Duchêne, *On the hydrostatic limit of stably stratified fluids with isopycnal diffusivity*, arXiv:2206.01058 (2022).

Elisabetta Chiodaroli

Dipartimento di Matematica, Università di Pisa

On Wild Initial Data for the Isentropic Euler System of Gas Dynamics

Abstract. In this talk we deal with the Cauchy problem for the isentropic Euler equations of gas dynamics in several space dimensions and we consider the set of wild data, that is initial data allowing for infinitely many weak solutions forward in time. We first prove the existence in 2D of smooth initial data which are wild, namely, they admit infinitely many bounded admissible weak solutions. Then we present a result of density of wild initial data in the energy space. The proofs rely on convex integration arguments à la De Lellis and Székelyhidi.

References

[1] E. Chiodaroli, O. Kreml, V. Macha, S. Schwarzacher, *Non–uniqueness of admissible weak solutions to the compressible Euler equations with smooth initial data*, Trans. Amer. Math. Soc., Vol. 374, 2021, pp. 2269-2295.

[2] E. Chiodaroli, E. Feireisl, *On the density of wild data to the isentropic Euler system of gas dynamics*, in preparation (2022).

GIUSEPPE MARIA COCLITE

Politecnico di Bari

Nonlocal regularization of conservation laws

Abstract. We present some recent results on the problem of approximating a scalar conservation law by a conservation law with nonlocal flux. As convolution kernel in the nonlocal flux, we consider an exponential-type approximation of the Dirac distribution. We prove that the (unique) weak solution of the nonlocal problem converges strongly in $C(L^1_{loc})$ to the entropy solution of the local conservation law.

This talk is based on joint works [1, 2] with J.-M. Coron, N. De Nitti, A. Keimer, and L. Pflug.

References

[1] G. M. Coclite, J.-M. Coron, N. De Nitti, A. Keimer, and L. Pflug, *A general result on the approximation of local conservation laws by nonlocal conservation laws: The singular limit problem for exponential kernels.*, Ann. Inst. H. Poincaré C Anal. Non Linéaire, (2022).

[2] G. M. Coclite, N. De Nitti, A. Keimer, and L. Pflug, *On existence and uniqueness of weak solutions to nonlocal conservation laws with BV kernels*, Z. Angew. Math. Phys., (2022).

Sara Daneri

Gran Sasso Science Institute

On the sticky particle solutions to the pressureless Euler system in general dimension

Abstract. In this talk we consider the pressureless Euler system in dimension greater than or equal to two. Several works have been devoted to the search for solutions which satisfy the following adhesion or sticky particle principle: if two particles of the fluid do not interact, then they move freely keeping constant velocity, otherwise they join with velocity given by the balance of momentum. For initial data given by a finite number of particles pointing each in a given direction, in general dimension, it is easy to show that a global sticky particle solution always exists and is unique. In dimension one, sticky particle solutions have been proved to exist and be unique. In dimensions greater than or equal to two, it was shown that as soon as the initial data is not concentrated on a finite number of particles, it might lead to non-existence or non-uniqueness of sticky particle solutions. In collaboration with S. Bianchini, we show that even though the sticky particle solutions are not well-posed for every measure-type initial data, there exists a comeager set of initial data in the weak topology giving rise to a unique sticky particle solution. Moreover, for any of these initial data the sticky particle solution is unique also in the larger class of dissipative solutions (where trajectories are allowed to cross) and is given by a trivial free flow concentrated on trajectories which do not intersect. In particular for such initial data there is only one dissipative solution and its dissipation is equal to zero. Thus, for a comeager set of initial data the problem of finding sticky particle solutions is well-posed, but the dynamics that one sees is trivial. Our notion of dissipative solution is lagrangian and therefore general enough to include weak and measure-valued solutions.

References

[1] S. Bianchini and S. Daneri *On the sticky particle solutions to the multi-dimensional pressureless Euler equations*, arXiv:2004.06557 (2020).

MICHELE DOLCE

Imperial College London

On maximally mixed equilibria of two-dimensional perfect fluids

Abstract. The motion of a two-dimensional incompressible and inviscid fluid can be described as an area-preserving rearrangement of the initial vorticity that preserves the kinetic energy. In the infinite time limit, some irreversible mixing can occur and predicting what structures can persist is an issue of fundamental importance. Shnirelman [2] introduced the concept of maximally mixed states (any further mixing would necessarily change their energy) and proved they are perfect fluid equilibria. We offer a new perspective on this theory by showing that any minimizer of any strictly convex Casimir, in a set containing Euler's end states, is maximally mixed. Thus, (weak) convergence to equilibrium cannot be excluded solely on the grounds of vorticity transport and conservation of kinetic energy. On the other hand, in the straight channel, we give examples of open sets of initial data which can be arbitrarily close to any shear flow in L1 of vorticity but do not weakly converge to them in the long time limit.

This is a joint work with T.D. Drivas [1].

References

[1] M. Dolce and T. D. Drivas, *On maximally mixed equilibria of two-dimensional perfect fluids*, arXiv:2204.03587 (2022).

[2] A. I. Shnirelman *Lattice theory and flows of ideal incompressible fluid*, Russ. J. Math. Phys, 1 (1993), pp. 105–113.

SIMONE FAGIOLI

DISIM, Università degli studi dell'Aquila

On a chemotaxis-hapotaxis system with nonlinear diffusion modelling multiple sclerosis

Abstract. In this talk we will present a chemotaxis-hapotaxis type model, introduced in [1], [3] for the dynamics of early stage multiple sclerosis. This model consists of three equations describing the evolution of macrophages (m), cytokine (c) and apoptotic oligodendrocytes (d). The system we are interested in is

$$\begin{cases} \partial_t m = \nabla \cdot (D(m)\nabla m - \chi f(m)\nabla c) + M(m), \\ \tau \partial_t c = \alpha \Delta c + \lambda d - c + \beta m, \\ \partial_t d = rh(m)\,(1-d)\,, \end{cases} \tag{1}$$

endowed with homogeneous Neumann boundary conditions in a bounded smooth domain $\Omega \subset \mathbb{R}^d, d = 1, 2, 3$. Here $\chi, \tau, \alpha, \lambda, \beta, r$ are given nonnegative parameters. The main novelty in our work is the presence of a nonlinear diffusivity $D(m)$, which results to be more appropriate from the modelling point of view. Under suitable assumptions and for sufficiently regular initial data, adapting the strategy in [4], [5], we show the existence of global bounded solutions for the model analysed. Finally, we will carry out some numerical simulations to show the disease pattern formation in $1-$d and $2-$d.

References

[1] V. Calvez, R. Khonsari, *Mathematical description of concentric demyelination in the human brain: self-organization models, from Liesegang rings to chemotaxis*, Math. Comput. Model. **47** (2008): 726 – 742.

[2] S. Fagioli, L. Romagnoli, *On a chemotaxis-hapotaxis model with nonlinear diffusion modelling multiple sclerosis*, preprint, (2022).

[3] R. Khonsari, V. Calvez, *The origins of concentric demyelination: self-organization in the human brain*, PLos ONE **2** (2007): e150.

[4] Y. Li, J. Lankeit, *Boundedness in a chemotaxis-haptotaxis model with nonlinear diffusion*, Nonlinearity **29** (2016), 1564 – 1595.

[5] Y. Tao and M. Winkler, *A chemotaxis-haptotaxis model: The roles of nonlinear diffusion and logistic source*, SIAM J. Math. Anal., **43** (2011), 685 – 704.

Mauro Garavello

Dipartimento di Matematica e Applicazioni, Università di Milano-Bicocca

Macroscopic traffic models and autonomous vehicles

Abstract. We present in this talk two models for controlling car traffic through special vehicles, like autonomous or connected ones. Car traffic is described by macroscopic models, while the dynamics of the special vehicles is described by a microscopic model. More precisely, either the Lighthill-Whitham-Richards (LWR) model [3, 4]

$$\partial_t \rho + \partial_x f(\rho) = 0 \tag{2}$$

or, alternative, the Colombo-Marcellini-Rascle (CMR) model [1]

$$\begin{cases} \partial_t \rho + \partial_x \left(\rho\, v(\rho, w)\right) = 0 \\ \partial_t \rho w + \partial_x \left(\rho w\, v(\rho, w)\right) = 0\,, \end{cases} \tag{3}$$

describes the evolution of traffic in a single road. Here $\rho = \rho(t, x)$ denotes the density of traffic at time t and at position x, $f : \mathbb{R} \to \mathbb{R}$ is the flux, $w = w(t, x)$ denotes the maximal speed of drivers, and $v = v(\rho, w)$ is the average speed. Assume that a vehicle (or more vehicles), whose position at time t is described by the function $y = y(t)$, aims at controlling the behavior of traffic. The evolution of such a vehicle is described by the ODE

$$\dot{y}(t) = u(t) \tag{4}$$

where $u = u(t)$ is a control function, which selects the desired speed.
Following the ideas proposed by Delle Monache and Goatin in [2], we consider two control models, one based on the LWR model (2) and one on the CMR model (3), where the control acts on the autonomous vehicle, through the equation (4). We discuss about the concepts of solutions for the two systems and we show that, given a control function $u = u(t)$ with finite total variation, a solution exists for both systems. The proofs are based on the wave-front tracking technique.

These are joint works with P. Goatin, T. Liard, F. Marcellini, and B. Piccoli.

References

[1] R.M. Colombo, F. Marcellini, M. Rascle. A 2-phase traffic model based on a speed bound. *SIAM J. Appl. Math.* 70(7):2652-2666, 2010.

[2] M. L. Delle Monache, P. Goatin. Scalar conservation laws with moving constraints arising in traffic flow modeling: an existence result. *J. Differential Equations*, 257(11):4015–4029, 2014.

[3] M. J. Lighthill, G. B. Whitham. On kinematic waves. II. A theory of traffic flow on long crowded roads. *Proc. Roy. Soc. London. Ser. A.*, 229:317–345, 1955.

[4] P. I. Richards. Shock waves on the highway. *Operations Res.*, 4:42–51, 1956.

GRAZIANO GUERRA

Dipartimento di Matematica e Applicazioni, Università degli Studi di Milano-Bicocca

Balance Laws with Singular Source Term and Applications to Fluid Dynamics

Abstract. Conservation laws in one space dimension, i.e., systems of partial differential equations in conservative form of the type

$$\partial_t u + \partial_x f(u) = 0 \qquad t \geq 0,\ x \in \mathbb{R}\,, \tag{5}$$

allow to describe, for instance, the movement of a fluid along a rectilinear pipe with constant section. When, at a point $\bar{x}$, the pipe's direction or its section changes, the defect in the conservation of the u variable, under suitable assumptions, can be modeled by a weight Ξ:

$$f\left(u(t,\bar{x}+)\right) - f\left(u(t,\bar{x}-)\right) = \Xi\big(z^+, z^-, u(t,\bar{x}-)\big) \quad \text{for a.e. } t > 0, \tag{6}$$

where z^+ and z^- identify the physical parameters that change across $\bar{x}$. The finite propagation speed, intrinsic to (5), allows then to extend to any finite number of points $\bar{x}_0, \bar{x}_1, \ldots, \bar{x}_k$ in the following way

$$\begin{cases} \partial_t u + \partial_x f(u) = \sum\limits_{i=1}^{k-1} \Xi\left(\zeta_k(\bar{x}_i+), \zeta_k(\bar{x}_i-), u(t,\bar{x}_i-)\right)\ \delta_{\bar{x}_i} \\ u(0,x) = u_o(x), \end{cases} \tag{7}$$

where the jumps in the flux are described by the sum of Dirac deltas δ_{x_i} centered at $x = x_i$ and weighted by Ξ. Here ζ_k is the piecewise constant function attaining the $k+1$ constant values $z_0, z_1, \ldots, z_k$ on the intervals $]-\infty, \bar{x}_1[$, $]\bar{x}_1, \bar{x}_2[$, $\ldots$, $]\bar{x}_k, +\infty[$.

We provide a detailed description of the rigorous limit $k \to +\infty$ of (7), covering its extension to the case of a general $\mathbf{BV}$ function $\zeta \in \mathbb{R}^p$ under the non resonance condition, i.e., we require that all eigenvalues of (5) be separated from 0.

References

[1] Rinaldo M. Colombo, Graziano Guerra, and Yannick Holle, *Non conservative products in fluid dynamics.* Nonlinear Anal. Real World Appl., Vol. 66 (2022) Paper No. 103539.

Giacomo Maria Leccese

Scuola Internazionale Superiore di Studi Avanzati, Trieste

On the sticky particle solutions to the multi-dimensional Hamiltonian system

Abstract. We consider a multi-dimensional Hamiltonian system of probability measures with finite quadratic momentum and we deal with existence and uniqueness of specific classes of solutions, that are sticky particle solutions and dissipative solutions. In the specific case of pressureless Euler system, in [1] there are explicit counterexamples to both existence and uniqueness of sticky solutions, however in [2] authors prove that for a comeager set of initial data in the weak topology the pressureless Euler system admits a unique sticky particle solution given by a free flow where trajectories are disjoint straight lines. In the same article authors introduce the broader class of dissipative solutions that decreasing their kinetic energy, which turns out to be the compact weak closure of the classical sticky particle solutions.

We provide a definition of sticky and dissipative solutions for the Hamiltonian system, where in particular there is an interaction between particles together with a condition of stickiness/dissipation. We also prove the existence of comeager set of initial data such that the Hamiltonian system admits a unique dissipative solution.

References

[1] A. Bressan , T. Nguyen, *Non-existence and non-uniqueness for multidimensional sticky particle systems*, Kinetic and related models 7 (2), 205–218 (2014).

[2] S. Bianchini, S. Daneri. *On the sticky particle solutions to the multi-dimensional pressureless Euler equations*, arXiv:2004.06557 (2020).

[3] S. Bianchini, G. Leccese, *On the sticky particle solutions to the multi-dimention Hamiltonian system*, in preparation.

ALESSANDRO MORANDO

Università degli Studi di Brescia

Local existence of 2D compressible current-vortex sheets

Abstract. We are concerned with the nonlinear characteristic free boundary problem for the existence of current-vortex sheets in ideal compressible Magneto-hydrodynamics in two space dimension. We first identify a sufficient condition ensuring the weak stability of the linearized current-vortex sheet problem. Then the local existence of the original nonlinear problem is proved in anisotropic Sobolev spaces, by using a suitable modification of Nash-Moser iteration scheme, provided that the stability condition above is satisfied at each point of the initial discontinuity front.

This is a joint work with Paolo Secchi, Paola Trebeschi and Difan Yuan.

Laura V. Spinolo

IMATI-CNR, Pavia

The singular local limit of nonlocal traffic models with general kernels

Abstract. The signature feature of nonlocal conservation laws is that the flux function depends on the solution through the convolution with a given kernel. In the last few years, these equations have been extensively studied owing to their wide range of applications, such as vehicular traffic models. In this talk I will address the singular local limit: when the the convolution kernel is replaced by a Dirac delta, the nonlocal equation formally boils down to a (classical) conservation law. Whether or not the solutions of the nonlocal equations converge to the entropy admissible solution of the limit conservation law is a question that was posed in [1]. Convergence in the general case is ruled out by the counter-examples in [6], but in the specific framework of traffic models (with anisotropic convolution kernels) the singular limit has been so far established under quite rigid assumptions, i.e. in the case of the exponential kernel [2, 3] or under fairly restrictive conditions on the initial datum [4]. In this talk I will discuss new results established in [5], namely (i) a general convergence theorem which holds under assumptions that are entirely natural in view of applications to traffic models, plus a convexity requirement on the convolution kernels; (ii) a general criterion for the entropy admissibility of the limit; (iii) a convergence rate; (iv) a counter-example showing that the convexity requirement on the convolution kernels is necessary to achieve the main compactness estimate.

References

[1] P. Amorim, R. M. Colombo and A. Teixeira, *On the numerical integration of scalar nonlocal conservation laws*, ESAIM Math. Model. Numer. Anal., 49(1):19–37, 2015.

[2] A. Bressan and W. Shen, *On traffic flow with nonlocal flux: a relaxation representation*, Arch. Ration. Mech. Anal., 237(3):1213–1236, 2020.

[3] G. M. Coclite, J.-M. Coron, N. De Nitti, A. Keimer, and L. Pflug, *A general result on the approximation of local conservation laws by nonlocal conservation laws: The singular limit problem for exponential kernels*, Ann. Inst. H. Poincaré C Anal. Non Linéaire, (2022).

[4] M. Colombo, G. Crippa, E. Marconi, and L. V. Spinolo, *Local limit of nonlocal traffic models: convergence results and total variation blow-up*, Ann. Inst. H. Poincaré C Anal. Non Linéaire, 38(5):1653–1666, 2021.

[5] M. Colombo, G. Crippa, E. Marconi, and L. V. Spinolo, *Nonlocal traffic models with general kernels: singular limit, entropy admissibility and convergence rate*, preprint ArXiv:2206.03949

[6] M. Colombo, G. Crippa, and L. V. Spinolo, *On the singular local limit for conservation laws with nonlocal fluxes*, Arch. Rat. Mech. Anal., 233(3):1131–1167, 2019.

STEFANO SPIRITO

DISIM, Università degli Studi dell'Aquila

Propagation of regularity and uniqueness for a Kelvin-Voigt model in viscoelasticity

Abstract. We consider nonlinear viscoelastic materials of Kelvin-Voigt type with stored energies satisfying the Andrews-Ball condition and a linear viscous stress. We show the existence of weak solutions with deformation gradients in H^1 for energies of any superquadratic growth. Moreover, we prove uniqueness in the case of two space dimensions.

This is joint work with K. Koumatos, C. Lattanzio, and A.E. Tzavaras.

Luca Talamini

Università degli Studi di Padova

Regularity and initial data identification for conservation laws with space discontinuous flux

Abstract. We consider the Cauchy problem for the scalar conservation law

$$u_t + f(u,x)_x = 0 \tag{8}$$

where f is a discontinuous function in the space variable:

$$f(u,x) = \begin{cases} f_l(u), & x < 0, \\ f_r(u), & x > 0. \end{cases}$$

Here f_l, f_r are strictly convex, smooth, maps. The discontinuity of the flux naturally leads to the study of infinitely many L^1 contractive semigroups $\mathcal{S}_t^{AB}$, each one associated to particular pair of values, a *connection*, (A,B) (that detemine a flux contraint on the solutions at $x=0$) . The solution u associated to (A,B) will be the unique one that dissipates the additional generalized Kružkov entropy

$$\eta^{AB} = \begin{cases} |u-A|, & x < 0, \\ |u-B|, & x > 0. \end{cases}$$

In general solutions of (8) do not have bounded total variation near the interface $x=0$. Motivated by the study of controllability properties, in the first part of the talk we shall discuss some adapted Oleinik estimates, which require a detailed analysis of the local structure of solutions near the interface $x=0$.

The second part of the talk will be devoted to the problem of initial data identification. Namely, for $\omega \in L^\infty$, our goal will be to characterize the set

$$\mathcal{I}_T^{AB}\omega = \{u_0 \in \mathbb{L}^\infty \ : \ \mathcal{S}_T^{AB} u_0 = \omega\}.$$

In particular, we shall prove that, whenever $\mathcal{I}_T^{AB}\omega \neq \emptyset$, the set $\mathcal{I}_T^{AB}\omega$ is either a singleton, or an infinite dimensional (non convex) cone. Finally, we shall define an appropriate notion of backward operator through which we characterize the vertex of the cone $\mathcal{I}_T^{AB}\omega$.

This is a joint research with Fabio Ancona (Università di Padova).

References

[1] F. Ancona, M. T. Chiri, *Attainable profiles for conservation laws with flux function spatially discontinuous at a single point*, ESAIM: COCV, Vol. 26, (2020), pp 1–33.

[2] F. Ancona, M. T. Chiri, L. Talamini, *Correction to: Attainable profiles for conservation laws with flux function spatially discontinuous at a single point*, in preparation.

[3] F. Ancona, L. Talamini, *On the attainable set for conservation laws with space discontinuous flux and time dependent connections*, in preparation.

[4] F. Ancona, L. Talamini, *Initial data identification for conservation laws with space discontinuous flux*, in preparation.

MARTINA ZIZZA

SISSA, Trieste

Properties of Mixing BV vector fields

Abstract. We consider the measure space $(K, \mathcal{B}(K), \mathcal{L}^2)$ where K is the unit square $[0,1]^2$, $\mathcal{B}(K)$ is the Borel σ-algebra and $\mathcal{L}^2$ denotes the normalized Lebesgue measure. A map $T : K \to K$ is an *automorphism* of the unit square (in symbols, $T \in G(K)$) if it is invertible and measure-preserving, that is

$$\mathcal{L}^2(T^{-1}(A)) = \mathcal{L}^2(A), \quad \forall A \in \mathcal{B}(K). \tag{9}$$

An automorphism $T \in G(K)$ is a *permutation* if it permutes the subsquares of some grid $\mathbb{N} \times \mathbb{N}\frac{1}{n}$ of the unit square K, that is the grid made of subsquares of side $\frac{1}{n}$, for some $n \in \mathbb{N}$.

Among automorphisms, permutations play an important role in Ergodic Theory, since they are used to prove some density properties for mixing automorphisms, namely:

Definition Let $T \in G(K)$ be an automorphism. We say that T is mixing if $\forall A, B \in \mathcal{B}(K)$,

$$\lim_{n\to\infty} \mathcal{L}^2(T^{-n}(A) \cap B) = \mathcal{L}^2(A)\mathcal{L}^2(B).$$

In this talk we will provide a parallelism between Fluid Dynamics and Ergodic Theory defining permutation vector fields, that is divergence-free vector fields $b \in L^\infty([0,1], \mathrm{BV}(K))$, whose flow $X_{t=1} : K \to K$, when evaluated at time $t = 1$, is a permutation of subsquares of K.
We will first prove that divergence-free vector fields $b \in L^\infty([0,1], \mathrm{BV}(K))$ can be approximated by permutation vector fields, using an argument introduced by Shnirelman in [1], then we will show how these vector fields can be manipulated in order to obtain mixing vector fields, namely divergence-free vector fields $b \in L^\infty_t \mathrm{BV}_x$ whose flow $X_{t=1}$, when evaluated at time $t = 1$, is a mixing automorphism of K.

References

[1] A.I. Shnirelman, *The geometry of the group of diffeomorphisms and the dynamics of an ideal incompressible fluid*, Math. USSR Sb., Vol. 56, (1985), pp 82-109.

[2] S. Bianchini, M. Zizza, *Properties of Mixing BV vector fields*, Preprint (2021).